and only if, the term in the bracket vanishes. Evaluating the second derivative at $dI/du_2 = 0$ yields

$$\frac{d^2I}{du_2^2} = -F(h_2 - u_2)\left\{u_2\left[\frac{h_2}{(2 + 2E_2 - u_2^2)^{1/2}} - \frac{u_2}{(h_2^2 - u_2^2)^{1/2}}\right] + (h_2^2 - u_2^2)^{1/2}\right\}$$

We conclude that $(d^2I/du_2^2) < 0$, and if any extreme of I exists, it must be a maximum. Therefore, the minimum value of I occurs at the extremes of u_2, either at the perigee or at infinity.

The value of I at infinity (I_∞) must be compared with the value of I at the perigee (I_p) to see which is less.

Using the substitutions $a = (e_2^2 - 1)^{1/2}/p_2$ and $b = (e_2 + 1)/p_2$, where $0 < a < b < 1$, one obtains from Eq. (12),

$$I_p = (2)^{1/2}\left[\frac{1 - b}{(1 + b)^{1/2}} + \frac{b(b)^{1/2}}{(b^2 - a^2)^{1/2}}\right] \quad (13)$$

and

$$I_\infty = (2)^{1/2}\left[\left(\frac{1 - a}{1 + a}\right)^{1/2} + \frac{a(b)^{1/2}}{(b^2 - a^2)^{1/2}}\right] \quad (14)$$

We see from Eqs. (13) and (14) that as $a \rightarrow 0$, $I_p > I_\infty$, and as $a \rightarrow b$, $I_\infty > I_p$. Therefore, there is some value of a as a function of b, such that $I_p = I_\infty$. Setting (13) equal to (14), one obtains

$$-\frac{(1 - b)^{1/2}(1 - b)^{1/2}}{(1 + b)^{1/2}} + \left(\frac{1 - a}{1 + a}\right)^{1/2} = (b)^{1/2}\left(\frac{b - a}{b + a}\right)^{1/2}$$

Using the substitutions $x^2 = (1 - a)/(1 + a)$ and $y^2 = (1 - b)/(1 + b)$, we obtain a fourth degree equation in x, for which the only real positive solution is $x = [-b + (b^2 + b + 1)^{1/2}]/(1 + b)^{1/2}$; hence, $a = [b/(b^2 + b + 1)^{1/2}]$ for $I_p = I_\infty$.

Case 1. If $a > b/(b^2 + b + 1)^{1/2}$, then the optimum transfer occurs at the perigee and our problem is completely solved.

Case 2. If $a < b/(b^2 + b + 1)^{1/2}$, then the optimum transfer would be at infinity. For this case we are interested in the radial distance ρ^* for which the total impulse (I_ρ^*) of transfer is equal to the total impulse of transfer at the perigee (I_p).

From Eq. (12) we obtain I_ρ^*:

$$\frac{1}{2^{1/2}} I_\rho^* = \left[\frac{k - v}{k + v}\right]^{1/2} [1 + k^2a^2 - v^2]^{1/2} + v$$

where $v^2 = [1/\rho^* + ba^2/(b^2 - a^2)]$ and $k^2 = b/(b^2 - a^2)$.

The equation, $I_\rho^* = I_p$, becomes

$$v^2\left(2k - \frac{2}{2^{1/2}} I_p\right) + v\left(\frac{I_p^2}{2} - \frac{2}{2^{1/2}} I_p k + k^2a^2 + 1\right) + \left(\frac{kI_p}{2^{1/2}} - k - k^3a^2\right) = 0$$

The factor $v - k$ has been omitted for $v \leq bk < k$.

Since $1/\rho^* = b$ or $v = bk$ must be a solution, we obtain the other solution as $v = 1/(1 + b)^{1/2}$, which gives

$$\rho^* = \frac{(1 + b)(b^2 - a^2)}{b^2 - a^2(1 + b + b^2)}$$

Therefore, a transfer to the hyperbola at any $\rho > \rho^*$ requires less total impulse than a transfer at the perigee.

Another interesting quantity is the value of a as a function of b, such that a transfer at any point on the hyperbola is more economical than a transfer at the perigee. This is found by setting ρ^* equal to the perigee ($1/b$) and obtaining $a = b(1 - b - b^2)^{1/2}$. Therefore, if $a \leq b(1 - b - b^2)^{1/2}$, then a tangential transfer to any point on the hyperbola is more economical than a tangential transfer to the perigee.

Conclusion

It was found that the total impulse required is a minimum if the launching is done in the following manner:

1) Tangential launching from the planet.

2) The transfer trajectory is tangent to the hyperbolic orbit at the firing of the second impulse.

3a) For $a > [b/(b^2 + b + 1)^{1/2}]$, the second impulse should be fired at the perigee, where a denotes the ratio of the radius of the planet to the radial distance to the asymptote of the hyperbolic orbit, and b denotes the ratio of the radius of the planet to the radical distance to the perigee of the hyperbolic orbit.

3b) For $a < [b/(b^2 + b + 1)^{1/2}]$) the second impulse should be fired at infinity. For this case, if the second impulse is fired at any radial distance $\rho > [(1 + b)(b^2 - a^2)/b^2 - a^2(1 + b + b^2)]$, the transfer is more economical than if the second impulse is fired at the perigee. In addition, if $a \leq b(1 - b - b^2)^{1/2}$, then a transfer at any point on the final hyperbola is more economical than a transfer at the perigee.

Since we have a tangential launching, the value of the total impulse for a transfer from a circular to a hyperbolic orbit (I_{tr}) is related to that of the launching (I_{LN}) by the equality $I_{tr} = I_{LN} - 1$. Therefore, min I_{tr} = min $I_{LN} - 1$. This indicates that the results of his analysis can also be applied to the transfer from a circular to a nonintersecting hyperbolic orbit.

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Supersonic Transport Climb Path Optimization Including a Constraint on Sonic Boom Intensity

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The ability to compute minimum fuel acceleration-climb paths subject to prescribed sonic boom intensity limits for a representative supersonic transport design is demonstrated with a particular gradient penalty function technique. Numerical studies of these paths indicate a significant loss in range performance of approximately 500 statute miles when a conservative sonic boom intensity limit of 2.25 lb/ft² is observed.

Introduction

THE optimization of acceleration-climb paths for minimum fuel performance for the supersonic transport is regarded as a critical economic factor in the operation of these vehicles. Adherence to fuel-optimal paths however, intro-

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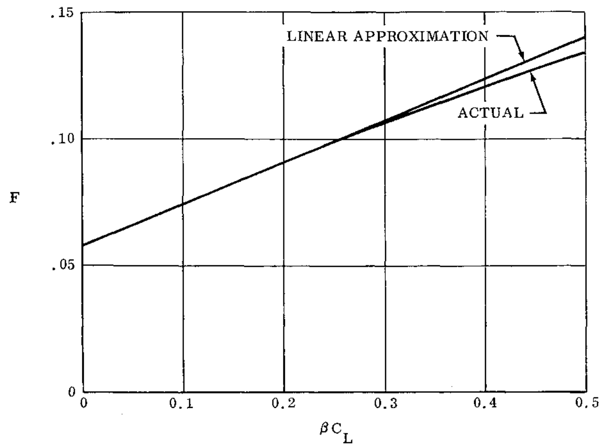

Fig. 1 The function $F(\beta C_L)$

duces an operational problem of high "sonic boom" intensities experienced along the ground track of the vehicle. The problem may then be stated: How would one modify the fuel-optimal flight path to yield minimum fuel performance loss while satisfying an imposed intensity limit everywhere along the ground track?

A computational technique for treating this type of problem is the gradient penalty function technique originally developed in Ref. 1 and subsequently applied to air vehicle trajectory optimization problems with terminal state equality and simple state variable inequality constraints.[2] The present note extends the technique in Ref. 2 by selecting another integral form of penalty function for treatment of a prescribed ground overpressure inequality constraint. The simplified overpressure formula to be utilized considers the lift and volume contributions of the vehicle and follows a *steady flight* analysis stemming from the classical theory of Whitham,[3] and later extended to a lifting wing-body combination by Walkden.[4] An approximation to the formula given in Ref. 4, experimentally verified by Carlson,[5] for thin delta wing configurations at small angles of attack, is then taken as the simplified overpressure formula for the optimization calculations.

Trajectory Equations

For flight in a fixed plane about a nonrotating earth we have the following system of differential equations describing the state variables:

Radial Acceleration

$$\dot{x}_1 = \dot{u} = \frac{v^2}{r} - g_e\left(\frac{r_e}{r}\right)^2 + g_e\frac{T}{W}\sin\theta - g_e\frac{D}{W}\sin\gamma + g_e\frac{L}{W}\cos\gamma \quad (1)$$

Circumferential Acceleration

$$\dot{x}_2 = \dot{v} = -\frac{uv}{r} + g_e\frac{T}{W}\cos\theta - g_e\frac{D}{W}\cos\gamma - g_e\frac{L}{W}\sin\gamma \quad (2)$$

Radius Rate

$$\dot{x}_3 = \dot{r} = u \quad (3)$$

Range Rate

$$\dot{x}_4 = \dot{\Phi} = \frac{v}{r} \quad (4)$$

Fuel Flow Rate

$$\dot{x}_5 = \dot{W} = -Q \quad (5)$$

For simplicity, a single control variable θ, attitude angle, is utilized with $\theta = \gamma + \alpha$. Here γ and α denote the vehicle flight-path angle and angle of attack, respectively. The wing geometry of the vehicle is fixed and full throttle operation assumed. The applied forces and fuel flow are generally represented in the following way:

$$L = f_1(M,h,\alpha)$$
$$D = f_2(M,h,\alpha)$$
$$T = f_3(M,h)$$
$$Q = f_4(M,h)$$

Gradient Penalty Function Technique Application

The gradient or steepest descent successive approximation technique for trajectory optimization with terminal state equality constraints, $(x_{if} - \bar{x}_i) = 0$ is presented in full detail in Ref. 1, and for the air vehicle minimum fuel application in Ref. 2. Briefly, for the minimum fuel problem with only terminal constraints we seek to minimize P', an approximation to P, where $P = -W_f$, with

$$P' = P + \frac{K_u}{2}(u_f - \bar{u})^2 + \frac{K_v}{2}(v_f - \bar{v})^2 + \frac{K_r}{2}(r_f - \bar{r})^2 + \frac{K_\Phi}{2}(\Phi_f - \bar{\Phi})^2 \quad (6)$$

Here $K_i = 0$ if associated terminal value is open, and $K_i > 0$ if terminal state value is specified. With the performance criterion and terminal constraints thus far being stated in terms of final values of the state variables, it then seems logical to introduce auxiliary variables with the violation of inequality constraints of the type $z_i \leq \bar{z}_i$ for $t_0 \leq t \leq t_f$ as numerical error estimates evaluated at the terminal point. This is accomplished by an integral form such as,

$$x_{if} = \frac{1}{2}\int_{t_0}^{t_f} (z_i - \bar{z}_i)^2 H(z_i - \bar{z}_i)dt \quad (7)$$

with the ultimate desire to have $x_{if} \to 0$ hence holding $z_i \leq \bar{z}_i$ for t_0 $t \leq \leq t_f$. Here $H(z_i - \bar{z}_i)$ is the unit step function. The equivalent first order differential equation governing the auxiliary variable $xl(t)$ becomes

$$\dot{x}_i = \tfrac{1}{2}(z - \bar{z})^2 H(z - \bar{z}) \qquad \text{with } x_i(t_0) = \text{const} \quad (8)$$

which is then integrated along with Eqs. (1–5) as an additional equation of state. If one treats the inequality constraints in this manner, the auxiliary variables dealing with minimum altitude, engine/airframe limits (given as airspeed/altitude limits), and ground overpressure limits are, respectively,

$$\dot{x}_6 = \tfrac{1}{2}(r - \tilde{r})^2 H(\tilde{r} - r) \qquad \text{with } x_6(t_0) = 0 \quad (9)$$

$$\dot{x}_7 = \tfrac{1}{2}[V(h) - \tilde{V}(h)]^2 H[V(h) - \tilde{V}(h)] \text{ with } x_7(t_0) = 0 \quad (10)$$

$$\dot{x}_8 = \tfrac{1}{2}(\Delta p_g - \Delta\tilde{p}_g)^2 H(\Delta p_g - \Delta\tilde{p}_g) \text{ with } x_8(t_0) = 0 \quad (11)$$

The modified performance index now becomes

$$P' = -W_f + \sum_{i=1}^{4} \frac{K_i}{2}(x_{if} - \bar{x}_i)^2 + \sum_{j=6}^{8} K_j x_{jf} \quad (12)$$

and for increasingly large values of K_i, K_j the solution of min P' will approach that of min P subject to the constraints $(x_{if} - \bar{x}_i) = 0$ at t_f and $z_j \leq \tilde{}$ for $t_0 \leq t \leq t_f$.

The Simplified Overpressure Formula

Carlson,[5] following the paper of Walkden,[4] has developed the following steady-flight approximation for the peak ground overpressure increment assuming the vehicle configuration to be composed of a slender body of revolution and a thin delta wing operating at small angles of attack:

$$\frac{\Delta p_g}{p_{\text{ref}}} = K_1(M^2 - 1)^{1/8} l^{3/4} h^{-3/4} [F(\beta C_L)] \quad (13)$$

Table 1 Trajectory terminal values

Trajectory	A	B	C	D	E	F
$\Delta\bar{P}_g$ lb/ft²	Not specified	Not specified	3.00	2.75	2.50	2.25
W_f lb	346,000	343,000	339,000	334,000	326,000	299,000
t_f sec	690	730	920	1090	1350	2350
ϕ_f rad	0.0525	0.0618	0.0719	.0835	0.1000	0.1610

where $p_{ref} = (p_h p_g)^{1/2}$ is taken as the geometric mean of ambient pressures. Here K_1 is a reflection factor, l the body length, $\beta = (M^2 - 1)^{1/2}$, and $F(\beta C_L)$ is determined by lift and cross-sectional area distribution considerations of the wing-body combination. One then seeks to refine Eq. (13) to account roughly for the effect of complete refraction of a sound ray due to a linear temperature gradient existing in the troposphere and of vehicle flight-path angle. Introducing Snell's law results in the formula

$$\mu + \gamma = (\pi/2) - \cos^{-1}(a_h/a_g) \tag{14}$$

where $\mu = \sin^{-1}(1/M)$. One may interpret Eq. (14) as a definition of "cut-off Mach number" M^* defined by h and γ as tabularly represented by Patterson.[6] That is, if $M > M^*(h,\gamma)$, then Δp_g is computed by Eq. (13); or if $M \leq M^*(h,\gamma)$, then $\Delta p_g = 0$. Our formula for Δp_g then becomes

$$\Delta p_g = (p_h p_g)^{1/2} K_1 (M^2 - 1)^{1/8} l^{3/4} h^{-3/4} [F(\beta C_L)] H(M - M^*) \tag{15}$$

Numerical Studies of Supersonic Transport Minimum Fuel Acceleration-Climb Paths

The results presented in this section are typical numerical solutions to the problem of minimum-fuel acceleration climb from post-takeoff initial conditions to begin-cruise terminal conditions for a representative canard/delta transport configuration. The $F(\beta C_L)$ function computed for this configuration is shown in Fig. 1. For this particular configuration, F is closely approximated as a linear function of βC_L. The initial conditions (held fixed) for Eqs. (1–5) are: $u_0 = 0$, $v_0 = 670$ fps, $h_0 = 500$ ft, $\Phi_0 = 0$, and $W_0 = 400{,}000$ lb. The desired terminal conditions are: $\bar{u} = 0$, $\bar{v} = 2900$ fps, $\bar{h} = 65{,}000$ ft, and $\bar{\Phi}$ open. The minimum fuel trajectory viewed in the Mach number-altitude plane (Fig. 2) without any inequality constraints is curve A. The accompanying overpressure history is shown in Fig. 3. The trajectory labeled B illustrates satisfaction of a representative engine/airframe limit defined by Eq. (10) without regard to overpressure limitations. Trajectories C through F are those paths satisfying particular overpressure limits without engine/airframe limit constraints. The accompanying table serves as a guide in determining the overpressure constraint limits treated in trajectories C through F and illustrates the strong performance tradeoffs computed for lowered sonic boom intensity limits. The overpressure limit of 2.25 lb/ft² (although still above a desired operational limit of 2.0 lb/ft²) in trajectory F is near the lowest attainable with the vehicle considered.

From trajectories B and F in Table 1, one may extract the following important quantitative result; the 44,000 lb increase in fuel expenditure due to satisfying a reasonable sonic boom intensity constraint (trajectory F) over that given by a path which disregards sonic boom considerations (trajectory B) becomes the weight equivalent of the vehicle payload. Alternately, the net-range loss experienced in using F compared to B is approximately 500 statute miles. (The range comparison is effected by an extension of trajectory B to include a cruise leg consuming fuel at approximately 50 lb/statute mile until the total fuel expenditure is equivalent to that of trajectory F.)

Conclusions

It becomes quite apparent from the foregoing results that careful acceleration climb-path programming with sonic boom intensity constraints will be a major factor governing the over-all economy of operation of this type of vehicle. To achieve this programming, an accurate means of predicting ground overpressure as a function of the trajectory variables, vehicle configuration (with its possible alteration during flight), and atmospheric properties is surely required. In spite of the present simplicity of the overpressure formula utilized here, the technique employed for optimal-path determination subject to practical-constraint considerations should assist the vehicle/power plant designer and performance analyst in more carefully predicting the operational economy of a particular vehicle.

At the time of writing, this author became aware of the study of Friedman[7] which presents a general treatment of the shock propagation problems. The quantitative effects of vehicle-linear acceleration and ray path length in the over-

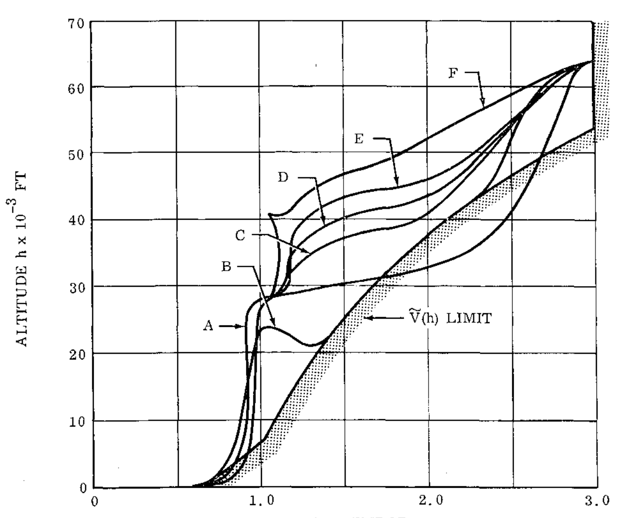

Fig. 2 Minimum fuel acceleration-climb profiles.

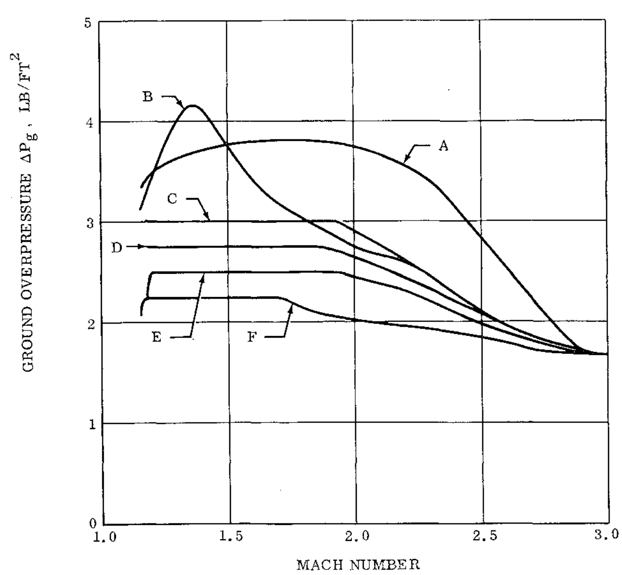

Fig. 3 Overpressure histories.

pressure equation developed in Ref. 7 would seem to be of most significance to the performance analyst in more carefully approximating the ground overpressure experienced during the vehicle acceleration-climb.

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Out-of-Plane Perturbations of a Circular Satellite Orbit

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The method of differential coefficients as applied to trajectory prediction and guidance analysis for space missions hinges on the validity of the approximations of first-order error analysis.[1, 2] In applying the linearity assumption to the variation of parameters for space trajectories, the in-plane and out-of-plane parameters are conveniently separated because small variations in in-plane parameters produce no first-order effect on the out-of-plane parameters, and vice versa.

It is the purpose of this note to investigate the validity of the first-order approximation concerning the out-of-plane motion of a long-lifetime satellite. Using a circular orbit for simplicity, it is shown that the effect of small deviations in in-plane parameters on the statistics of the out-of-plane motion may become significant, indicating the need of considering higher-order terms in certain cases.

Based on the linear perturbation theory, the perturbations in position and velocity perpendicular to the standard orbital plane at any time t are given by

$$\left.\begin{aligned} \delta z(t) &= \frac{\partial z(t)}{\partial z_0}\,\delta z_0 + \frac{\partial z(t)}{\partial \dot{z}_0}\,\delta \dot{z}_0 \\ \delta \dot{z}(t) &= \frac{\partial \dot{z}(t)}{\partial z_0}\,\delta z_0 + \frac{\partial \dot{z}(t)}{\partial \dot{z}_0}\,\delta \dot{z}_0 \end{aligned}\right\} \tag{1}$$

where δz_0 and $\delta \dot{z}_0$ are small variations in the out-of-plane position and velocity at time t_0, respectively. The partial derivatives are evaluated along the standard trajectory. The quantities δz_0 and $\delta \dot{z}_0$ can be thought of, for instance, as random errors due to guidance error sources at the end of the powered-flight phase (or injection).

For the case of circular orbit, Eqs. (1) take the forms

$$\left.\begin{aligned} \delta z(t) &= \cos n(t - t_0)\delta z_0 + \frac{\sin n(t - t_0)}{n}\,\delta \dot{z}_0 \\ \delta \dot{z}(t) &= -n \sin n(t - t_0)\delta z_0 + \cos n(t - t_0)\delta \dot{z}_0 \end{aligned}\right\} \tag{2}$$

where $n = v/r = \text{const}$, the r and v being, respectively, the radial distance and speed on the standard circular orbit. Assuming that δz_0 and $\delta \dot{z}_0$ are statistically independent with zero means and variances σ_{z0}^2 and $\sigma_{\dot{z}0}^2$, the variances and covariance of $\delta z(t)$ and $\delta \dot{z}(t)$, the quantities of primary interest, are easily determined from the equations

$$\left.\begin{aligned} \sigma_z^2(t) &= \sigma_{z0}^2 \cos^2 n(t - t_0) + \frac{\sigma_{\dot{z}0}^2 \sin^2 n(t - t_0)}{n^2} \\ \sigma_{\dot{z}}^2(t) &= \sigma_{z0}^2 \sin^2 n(t - t_0) + \sigma_{\dot{z}0}^2 \cos^2 n(t - t_0) \\ \mu_{z\dot{z}}(t) &= \frac{1}{2}\left(-n\sigma_{z0}^2 + \frac{\sigma_{\dot{z}0}^2}{n}\right)\sin 2n(t - t_0) \end{aligned}\right\} \tag{3}$$

Although it is clear that pure variations in in-plane parameters produce no perturbations in the out-of-plane direction, it is of interest to consider their influence on the behavior of out-of-plane motion in the presence of small variations δz_0 and $\delta \dot{z}_0$. Hence, let us reconsider Eqs. (2), taking into account that the in-plane parameters r and v are also randomly distributed within a small range about their respective standard values due to, for instance, guidance errors at injection.

For the purpose of comparison, consider r and v as random variables with the assumption that they are correlated in such a way that circularity of the orbit is conserved, i.e., $N = v/r$ is a random constant (independent of time). Although this assumption limits the consideration of a very special class of orbits, it permits examination of the problem effectively in a sample fashion.

In the numerical calculation that follows, assume that $\delta z_0 = 0$ with probability one (deterministic), that $\delta \dot{z}_0$ has mean zero and variances $\sigma_{\dot{z}0}^2$, and that the distribution of N is a discrete one approximating a one-dimensional normal distribution. Write

$$N = n(1 + X) \tag{4}$$

where X is a dimensionless random variable with mean zero, and the probability of taking the values $\pm 3\epsilon$, $\pm 2\epsilon$, $\pm\epsilon$, and 0 are, respectively, 0.0060, 0.0606, 0.2417, and 0.3834. The standard deviation of X is 0.052, with $\epsilon = 0.05$. Further assume that X and $\delta \dot{z}_0$ are statistically independent.

Equations (3) now have the forms, according to linear perturbation approximation,

$$[n^2/\sigma_{\dot{z}0}^2]\sigma_z^2(t) = \sin^2 n(t - t_0) \tag{5a}$$

$$[1/\sigma_{\dot{z}0}^2]\sigma_{\dot{z}}^2(t) = \cos^2 n(t - t_0) \tag{5b}$$

$$[n/\sigma_{\dot{z}0}^2]\mu_{z\dot{z}}(t) = \tfrac{1}{2}\sin 2n(t - t_0) \tag{5c}$$

By including in-plane variations, they become

$$\left[\frac{n^2}{\sigma_{\dot{z}0}^2}\right]\sigma_z^2(t) = \sum_j \left\{\frac{p_j \sin^2[(1 + X_j)n(t - t_0)]}{(1 + X_j)^2}\right\} \tag{6a}$$

$$\left[\frac{1}{\sigma_{\dot{z}0}^2}\right]\sigma_{\dot{z}}^2(t) = \sum_j p_j \cos^2[(1 + X_j)n(t - t_0)] \tag{6b}$$

$$\left[\frac{n}{\sigma_{\dot{z}0}^2}\right]\mu_{z\dot{z}}(t) = \sum_j \left\{\frac{p_j \sin[2(+ X_j)n(t - t_0)]}{2(1 + X_j)}\right\} \tag{6c}$$

where p_j is the probability of X taking the value X_j.

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